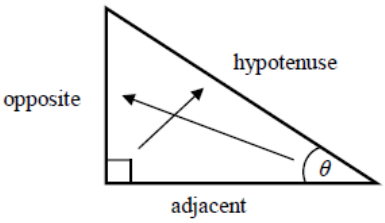
# Trigonometry in right-angled triangles

**A LEVEL LINKS**
**Scheme of work:** 4a. Trigonometric ratios and graphs

## Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle $\theta$ is called the opposite
  - the side next to the angle $\theta$ is called the adjacent.




- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle $\theta$, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle $\theta$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle $\theta$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

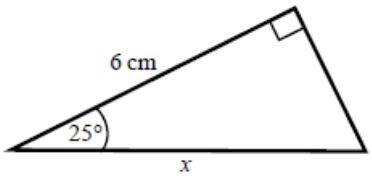
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

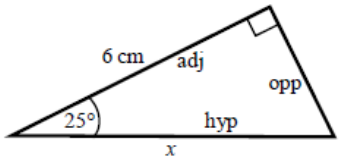
- The sine, cosine and tangent of some angles may be written exactly.

| | **0** | **30°** | **45°** | **60°** | **90°** |
|---|---|---|---|---|---|
| **sin** | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| **cos** | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| **tan** | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | |

# Examples

**Example 1** Calculate the length of side $x$.
Give your answer correct to 3 significant figures.




$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\cos 25° = \dfrac{6}{x}$

$x = \dfrac{6}{\cos 25°}$

$x = 6.620\,267\,5...$

$x = 6.62$ cm

1 Always start by labelling the sides.

2 You are given the adjacent and the hypotenuse so use the cosine ratio.

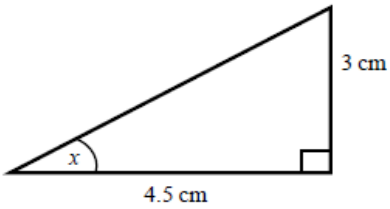
3 Substitute the sides and angle into the cosine ratio.

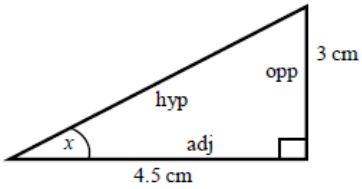
4 Rearrange to make $x$ the subject.

5 Use your calculator to work out $6 \div \cos 25°$.

6 Round your answer to 3 significant figures and write the units in your answer.

**Example 2** Calculate the size of angle $x$.
Give your answer correct to 3 significant figures.




$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan x = \dfrac{3}{4.5}$

$x = \tan^{-1}\left(\dfrac{3}{4.5}\right)$

$x = 33.690\,067\,5...$

$x = 33.7°$

1 Always start by labelling the sides.

2 You are given the opposite and the adjacent so use the tangent ratio.

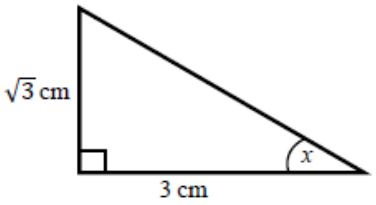
3 Substitute the sides and angle into the tangent ratio.

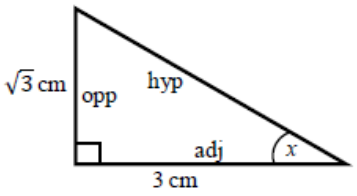
4 Use $\tan^{-1}$ to find the angle.

5 Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.

6 Round your answer to 3 significant figures and write the units in your answer.

**Example 3** Calculate the exact size of angle $x$.




$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan x = \frac{\sqrt{3}}{3}$$

$x = 30°$

**1** Always start by labelling the sides.

**2** You are given the opposite and the adjacent so use the tangent ratio.

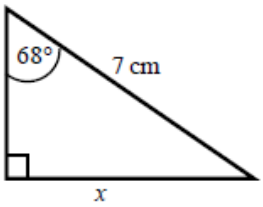
**3** Substitute the sides and angle into the tangent ratio.

**4** Use the table from the key points to find the angle.

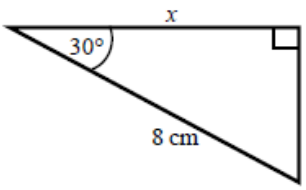
## Practice

**1** Calculate the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

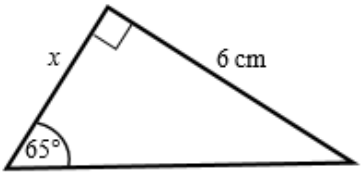
**a**



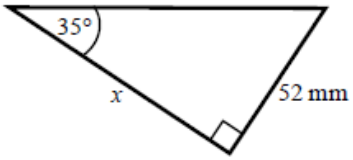

**b**



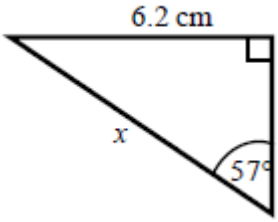

**c**



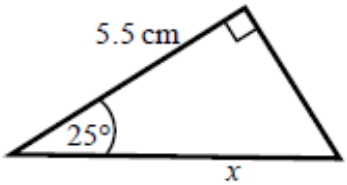

**d**




**e**

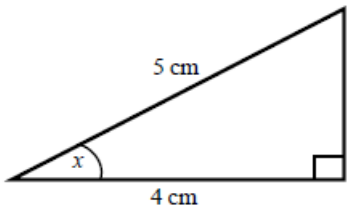



**f**

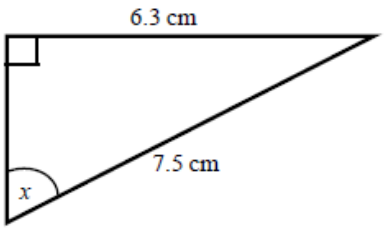


**2** Calculate the size of angle $x$ in each triangle.
Give your answers correct to 1 decimal place.

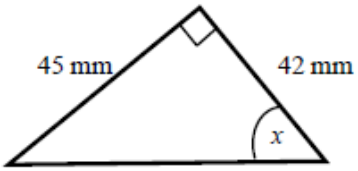
**a**



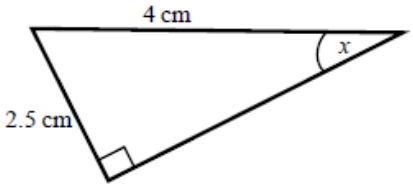

**b**




**c**

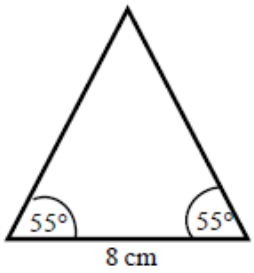



**d**



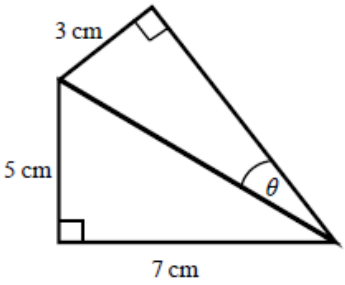

**3** Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

> **Hint:**
>
> Split the triangle into two right-angled triangles.



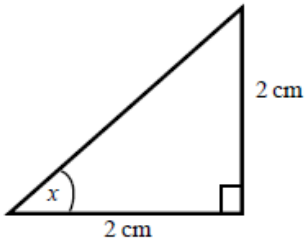

**4** Calculate the size of angle $\theta$.
Give your answer correct to 1 decimal place.

> **Hint:**
>
> First work out the length of the common side to both triangles, leaving your answer in surd form.

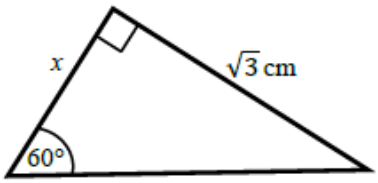



**5** Find the exact value of $x$ in each triangle.

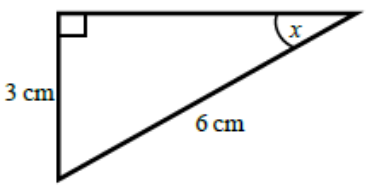
**a**



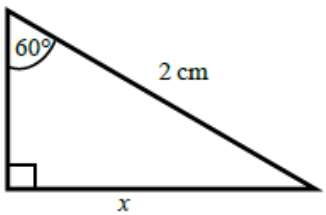

**b**




**c**




**d**

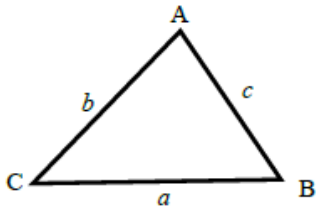


# The cosine rule

**A LEVEL LINKS**
**Scheme of work:** 4a. Trigonometric ratios and graphs
**Textbook:** Pure Year 1, 9.1 The cosine rule

## Key points

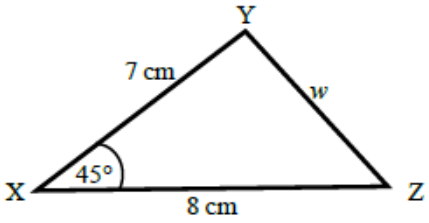
- $a$ is the side opposite angle A.
  $b$ is the side opposite angle B.
  $c$ is the side opposite angle C.

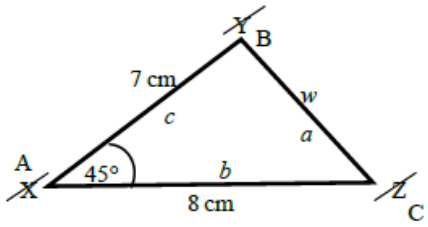



- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

## Examples

**Example 4** Work out the length of side $w$.
Give your answer correct to 3 significant figures.




$a^2 = b^2 + c^2 - 2bc\cos A$

$w^2 = 8^2 + 7^2 - 2\times 8\times 7\times \cos 45°$

$w^2 = 33.804\,040\,51...$
$w = \sqrt{33.804\,040\,51}$
$w = 5.81$ cm

**1** Always start by labelling the angles and sides.

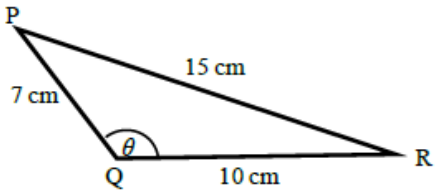
**2** Write the cosine rule to find the side.

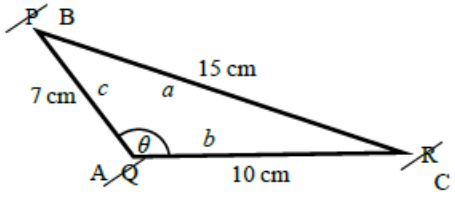
**3** Substitute the values $a$, $b$ and $A$ into the formula.

**4** Use a calculator to find $w^2$ and then $w$.

**5** Round your final answer to 3 significant figures and write the units in your answer.

**Example 5** Work out the size of angle $\theta$.
Give your answer correct to 1 decimal place.




$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$$

$$\cos\theta = \frac{-76}{140}$$

$\theta = 122.878\,349...$

$\theta = 122.9°$

1 Always start by labelling the angles and sides.

2 Write the cosine rule to find the angle.

3 Substitute the values $a$, $b$ and $c$ into the formula.

4 Use $\cos^{-1}$ to find the angle.

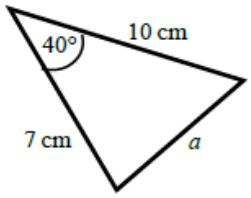
5 Use your calculator to work out $\cos^{-1}(-76 \div 140)$.

6 Round your answer to 1 decimal place and write the units in your answer.

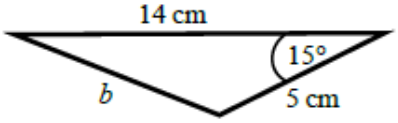
## Practice

**6** Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

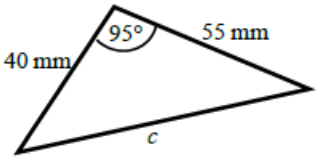
**a**



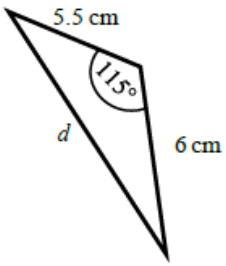

**b**




**c**

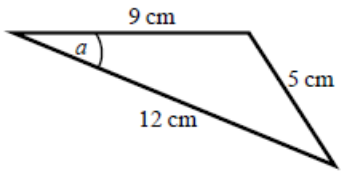



**d**

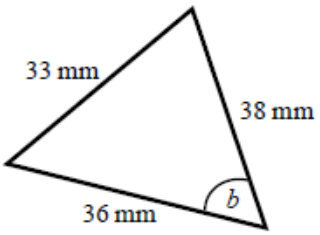


**7** Calculate the angles labelled $\theta$ in each triangle.
Give your answer correct to 1 decimal place.

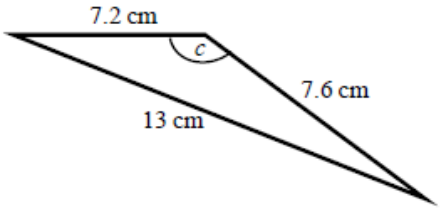
**a**



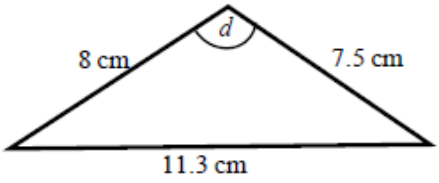

**b**




**c**

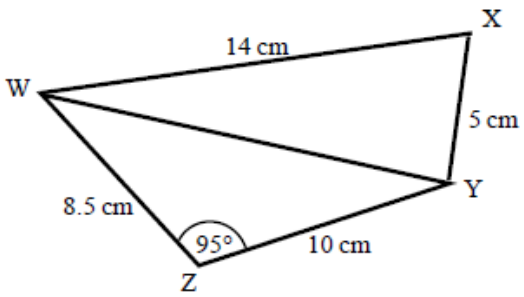



**d**




**8** **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

**b** Work out the size of angle WXY.
Give your answer correct to
1 decimal place.

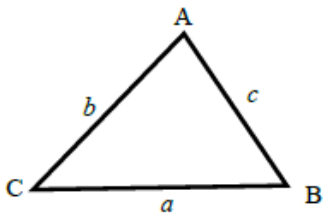


# The sine rule

**A LEVEL LINKS**
**Scheme of work:** 4a. Trigonometric ratios and graphs
**Textbook:** Pure Year 1, 9.2 The sine rule

## Key points

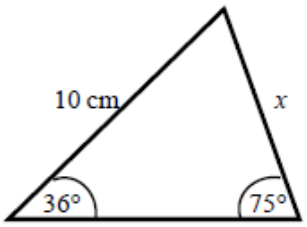
- $a$ is the side opposite angle A.
  $b$ is the side opposite angle B.
  $c$ is the side opposite angle C.

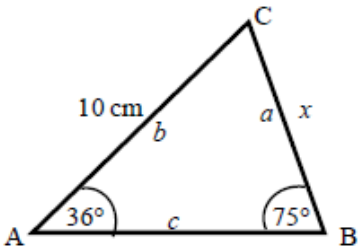



- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

## Examples

**Example 6** Work out the length of side $x$.
Give your answer correct to 3 significant figures.




$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$

$x = \frac{10 \times \sin 36°}{\sin 75°}$

$x = 6.09$ cm

**1** Always start by labelling the angles and sides.

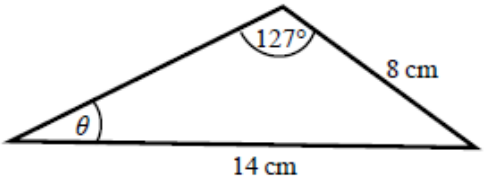
**2** Write the sine rule to find the side.

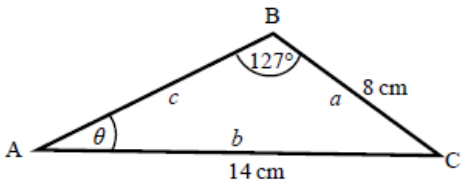
**3** Substitute the values $a$, $b$, $A$ and $B$ into the formula.

**4** Rearrange to make $x$ the subject.

**5** Round your answer to 3 significant figures and write the units in your answer.

**Example 7** Work out the size of angle $\theta$.
Give your answer correct to 1 decimal place.




$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin\theta}{8} = \frac{\sin 127°}{14}$$

$$\sin\theta = \frac{8 \times \sin 127°}{14}$$

$$\theta = 27.2°$$

1 Always start by labelling the angles and sides.

2 Write the sine rule to find the angle.

3 Substitute the values $a$, $b$, $A$ and $B$ into the formula.

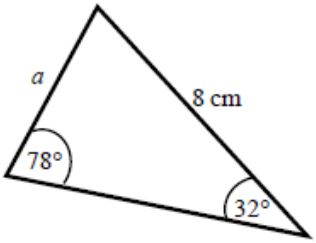
4 Rearrange to make $\sin\theta$ the subject.

5 Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

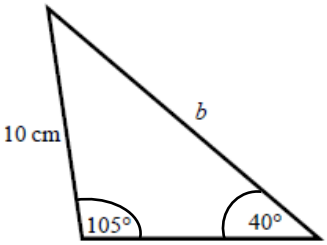
## Practice

**9** Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

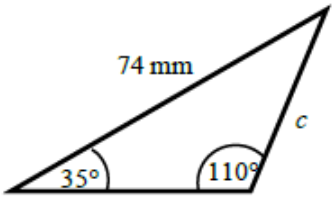
**a**



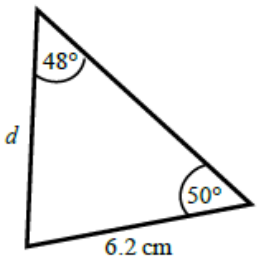

**b**




**c**

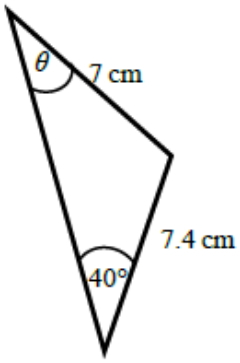



**d**

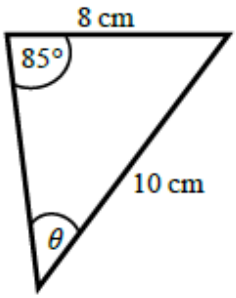


**10** Calculate the angles labelled $\theta$ in each triangle.
Give your answer correct to 1 decimal place.

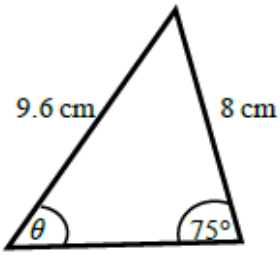
**a**



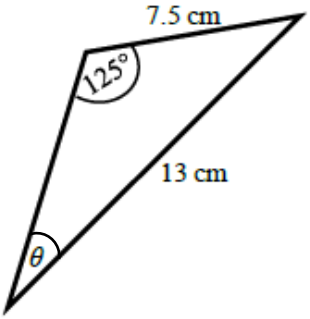

**b**




**c**

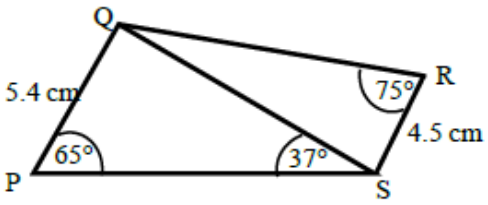



**d**




**11** **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

**b** Work out the size of angle RQS.
Give your answer correct to 1 decimal place.

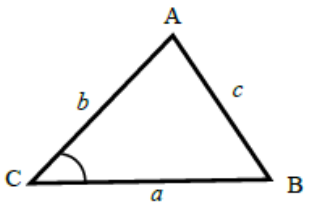


# Areas of triangles

**A LEVEL LINKS**
**Scheme of work:** 4a. Trigonometric ratios and graphs
**Textbook:** Pure Year 1, 9.3 Areas of triangles

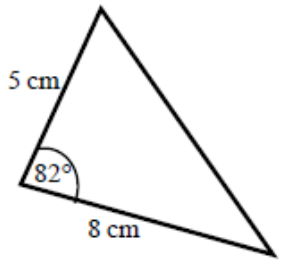
## Key points

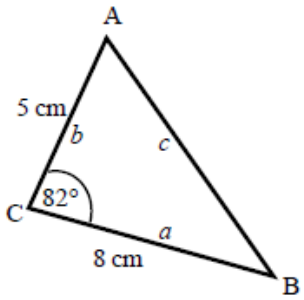
- $a$ is the side opposite angle A.
  $b$ is the side opposite angle B.
  $c$ is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.




## Examples

**Example 8** Find the area of the triangle.




Area = $\frac{1}{2}ab\sin C$

Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$

Area = 19.805 361...

Area = 19.8 cm$^2$

**1** Always start by labelling the sides and angles of the triangle.

**2** State the formula for the area of a triangle.

**3** Substitute the values of $a$, $b$ and $C$ into the formula for the area of a triangle.

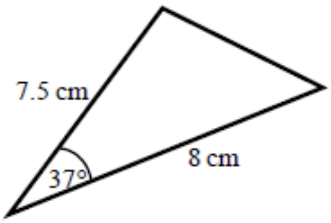
**4** Use a calculator to find the area.

**5** Round your answer to 3 significant figures and write the units in your answer.

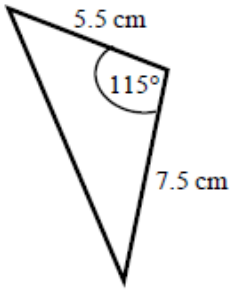
## Practice

**12** Work out the area of each triangle.
Give your answers correct to 3 significant figures.

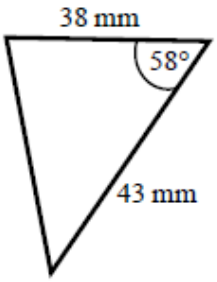
**a**



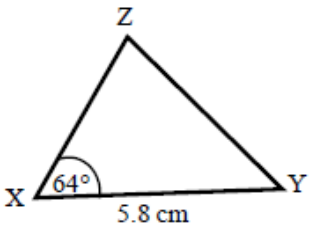

**b**




**c**




**13** The area of triangle XYZ is 13.3 $cm^2$.
Work out the length of XZ.



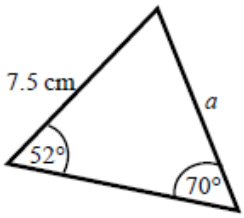

> **Hint:**
>
> Rearrange the formula to make a side the subject.

## Extend

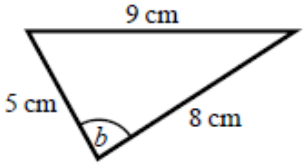
**14** Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

> **Hint:**
>
> For each one, decide whether to use the cosine or sine rule.

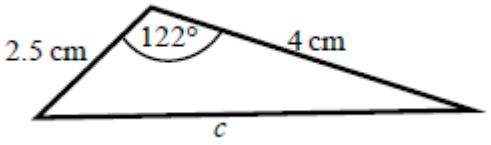
**a**



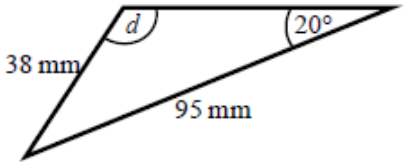

**b**



**c**




**d**




**15** The area of triangle ABC is 86.7 $cm^2$.
Work out the length of BC.
Give your answer correct to 3 significant figures.

